The swelling phenomenon of soils - Proposal of an efficient continuum modelling approach

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ABSTRACT: Under unconstrained conditions the (time dependant) swelling phenomenon of soils causes an immense increase in volume of the grain structure. Consequently, in regions where expansion is constrained, e.g. by structural elements, significant compression states are likely to develop.

Numerical treatment by means of the Finite Element method unfortunately tends to suffer from instable and oscillating behavior of the solution process. In contrast to commonly adopted "equivalent nodal force" approaches with the forces' magnitude being calculated on basis of the stress state prior to swelling, here, the authors propose an alternative – fully implicit – procedure, based on a continuum constitutive formulation for swelling. The swelling process is modeled within a viscous framework employing a backward-Euler integration scheme. The implicit viscous extension stabilizes the iterative solution process and visibly improves the global convergence behavior. Moreover, it enables the time dependant swelling characteristics to be simulated – if an appropriate calibration is provided.

1 CONSTITUTIVE MODELLING

1.1 *Final state constitutive relation*

Relevant for construction engineering purposes basically two swelling mechanisms can be distinguished, namely an "osmotic" and a "chemical" process. For both processes it could be shown that the increase in volume caused by swelling is dependent of the current stress state.

Based on load-controlled oedometric tests with multiple load steps (Huder-Amberg experiments) Grob (1972) proposed an approach that relates the axial final state swelling strains to the present compression stress state. Introduction of a compressive limit stress σ_c enables this basic equation to be extended to the full stress range. Further, by applying the 1D relation to the principal normal directions according to Wittke-Gattermann (1998), a generalized 3D relationship is obtained, reading

$$^{\infty}\varepsilon_{qi} = -k_q \cdot \begin{cases} 0 & , \quad \sigma_i \le \sigma_{0i} \\ \log\left(\sigma_i / \sigma_{0i}\right) & , \quad \sigma_{0i} < \sigma_i < \sigma_c \\ \log\left(\sigma_c / \sigma_{0i}\right) & , \quad \sigma_i \ge \sigma_c \end{cases} \qquad (1)$$

$k_q =$ swelling modulus

$\sigma_i =$ principal normal stress components of $\boldsymbol{\sigma}$ [1]

$\bar{\sigma}_{0i} =$ normal components of $\bar{\boldsymbol{\sigma}}_0$, $\bar{\boldsymbol{\sigma}}_0$ is obtained via transformation of $\boldsymbol{\sigma}_0$ to the principal directions of $\boldsymbol{\sigma}$; $\boldsymbol{\sigma}_0$ = equilibrium stress with respect to swelling (initial condition)

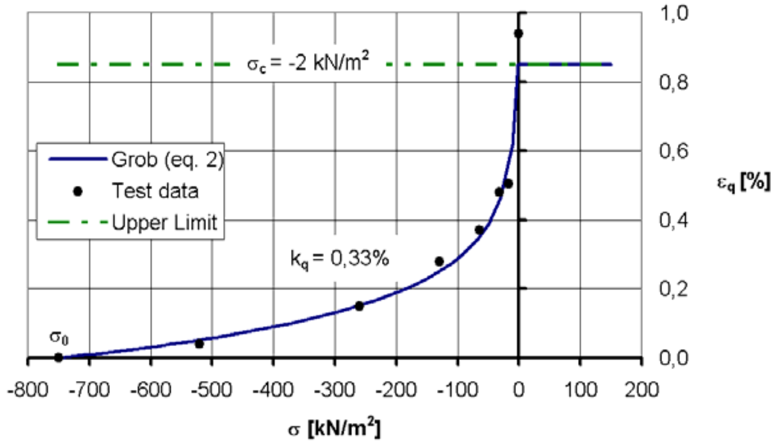

Figure 1. Constitutive one-dimensional relation for swelling according to Grob (1972)

For a more comprehensive discussion on the extension of Grob's basic equation, please refer to e.g. Wittke-Gattermann (1998), Heidkamp & Katz (2002).

1.2 *Modelling the time dependant strain evolution*

The constitutive equation reproduced above is limited to the final (stationary) state, i.e. it relates the developed swelling strains to the stress state that is present at time $t = \infty$. Subsequently, the relationship shall be extended to the time scale.

To achieve this, we start with a formal viscous approach. Correspondingly, consider the *rate of swelling strains* being defined as

$$\dot{\boldsymbol{\varepsilon}}_q = \frac{\boldsymbol{\varphi}(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_q)}{\eta} \quad ; \quad \boldsymbol{\varphi}(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_q) = {}^{\infty}\boldsymbol{\varepsilon}_q(\boldsymbol{\sigma}) - \boldsymbol{\varepsilon}_q \qquad (2)$$

with the *retardation time* η as a viscosity parameter and the vector valued *creep function* $\boldsymbol{\varphi}$. $\boldsymbol{\varepsilon}_q$ denotes the swelling strains that have developed at the considered time t .

In rheological terms this approach can be interpreted as a parallel coupling of a 'swelling' and a dashpot device.

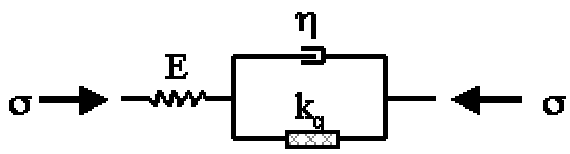

Figure 2. Principle sketch of one-dimensional rheological model for swelling

1.3 *Algorithmic treatment – implicit formulation*

In the framework of an incremental (time stepping) global solution procedure, the overall iteration algorithm yields a strain increment that the corresponding stress state must be computed for. In the

[1] Throughout this article tensile stresses and strains are considered as positive whereas compressive stresses and strains are negative.

following, an implicit algorithmic formulation for the constitutive relationship, that was described in the previous sections, is presented.

We start with the concept of additive strain decomposition. Accordingly, the total strain increment that relates to the time increment from $time = t$ to $time = t + \Delta t$ can be decomposed into elastic and inelastic contributions according to

$$\Delta\boldsymbol{\varepsilon} = \Delta\boldsymbol{\varepsilon}_e + \Delta\boldsymbol{\varepsilon}_q + \Delta\boldsymbol{\varepsilon}_{pl} \tag{3}$$

where $\Delta\boldsymbol{\varepsilon}_e$ and $\Delta\boldsymbol{\varepsilon}_q$ depend upon the – yet unknown – stress state ${}^{t+\Delta t}\boldsymbol{\sigma}$. Expressing the elastic strain increment in terms of the elastic compliance $\mathbf{C}$ and the stresses ${}^{t+\Delta t}\boldsymbol{\sigma}$, equation (3) is rewritten as

$$\mathbf{R}\left({}^{t+\Delta t}\boldsymbol{\sigma}, \Delta\boldsymbol{\varepsilon}_q\right) = \Delta\boldsymbol{\varepsilon} - \mathbf{C}\;\left({}^{t+\Delta t}\boldsymbol{\sigma} - {}^{t}\boldsymbol{\sigma}\right) - \Delta\boldsymbol{\varepsilon}_q - \Delta\boldsymbol{\varepsilon}_{pl} = \mathbf{0} \tag{4}$$

For the sake of clarity strain terms related to material non-linearity other than swelling are skipped in the subsequent notation. Furthermore, we restrict our treatment to isotropic material properties.

In order to integrate the swelling strain rates over the time increment we introduce the backward-Euler approach, resulting in

$$\Delta\boldsymbol{\varepsilon}_q = \int_t^{t+\Delta t} \frac{1}{\eta}\boldsymbol{\varphi}\left(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_q\right)\; d\tau \approx \frac{1}{\eta}\boldsymbol{\varphi}\left({}^{t+\Delta t}\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_q\right)\cdot \Delta t \tag{5}$$

where we have used ${}^{t+\Delta t}\boldsymbol{\varepsilon}_q = {}^{t}\boldsymbol{\varepsilon}_q + \Delta\boldsymbol{\varepsilon}_q$. Equivalently, we can write

$$\mathbf{r}\left({}^{t+\Delta t}\boldsymbol{\sigma}, \Delta\boldsymbol{\varepsilon}_q\right) = -\boldsymbol{\varphi}\left({}^{t+\Delta t}\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_q\right) + \frac{\eta}{\Delta t}\cdot\; \Delta\boldsymbol{\varepsilon}_q = \mathbf{0} \tag{6}$$

Equations (4) and (6) together with the constitutives (2) and (1) now define a nonlinear set of equations that we aim to solve for ${}^{t+\Delta t}\boldsymbol{\sigma}$ and $\Delta\boldsymbol{\varepsilon}_q$. Due to the inherent nonlinearity an iterative solution procedure is required. At some trial state k, $\mathbf{R}\big|_k = \mathbf{R}\left(\boldsymbol{\sigma}\big|_k, \Delta\boldsymbol{\varepsilon}_q\big|_k\right)$ and $\mathbf{r}\big|_k = \mathbf{r}\left(\boldsymbol{\sigma}\big|_k, \Delta\boldsymbol{\varepsilon}_q\big|_k\right)$ will have some value, probably not $\mathbf{0}$. An effective way of improving the trial is achieved by linearizing equations (4) and (6) and so arriving at a Newton-Raphson update scheme, that reads

$$\begin{vmatrix} \mathbf{C} & \mathbf{I} \\ \left.\dfrac{\partial\, {}^{\infty}\boldsymbol{\varepsilon}_q}{\partial\boldsymbol{\sigma}}\right|_k & -\left(\dfrac{\eta}{\Delta t} + \mathbf{I}\right) \end{vmatrix} \begin{vmatrix} \delta\boldsymbol{\sigma} \\ \delta\boldsymbol{\varepsilon}_q \end{vmatrix} = \begin{vmatrix} \mathbf{R}\big|_k \\ \mathbf{r}\big|_k \end{vmatrix} \tag{7}$$

$$\boldsymbol{\sigma}\big|_{k+1} = \boldsymbol{\sigma}\big|_k + \delta\boldsymbol{\sigma} \quad ; \quad \Delta\boldsymbol{\varepsilon}_q\big|_{k+1} = {}^{k}\,\Delta\boldsymbol{\varepsilon}_q\big|_k + \delta\boldsymbol{\varepsilon}_q \tag{8}$$

Due to the severe nonlinearity of the solution, the so obtained algorithm is enhanced by a line-search-like procedure, further motivation and treatment is given in Heidkamp & Katz (2002).

Note, that – not unexpected – for $\eta/\Delta t \to 0$ the iterative update (7) exactly reduces to the non-viscous case. To show this, quations (7) can be added, yielding

$$\left(\mathbf{C} + \left.\frac{\partial\, {}^{\infty}\boldsymbol{\varepsilon}_q}{\partial\boldsymbol{\sigma}}\right|_k\right)\delta\boldsymbol{\sigma} = \mathbf{R}\big|_k \tag{9a}$$

with $\mathbf{R}\big|_k$ now being given as

$$\mathbf{R}\big|_k = \Delta\boldsymbol{\varepsilon} - \underbrace{\mathbf{C}\left({}^{t+\Delta t}\boldsymbol{\sigma}\big|_k - {}^{t}\boldsymbol{\sigma}\right)}_{\Delta\boldsymbol{\varepsilon}_e|_k} - \underbrace{\left({}^{\infty}\boldsymbol{\varepsilon}_q\big|_k - {}^{t}\boldsymbol{\varepsilon}_q\right)}_{{}^{\infty}\Delta\boldsymbol{\varepsilon}_q|_k} \tag{9b}$$

Equations (9) represent the linearized form of the stationary (non-viscous) problem, given by equation (4) with $\Delta\boldsymbol{\varepsilon} = {}^{\infty}\boldsymbol{\varepsilon}_q(\boldsymbol{\sigma}) - {}^{t}\boldsymbol{\varepsilon}_q$ instead of (2). Consequently, if only the steady state solution is of interest, the viscous approach can be adopted to numerically smooth the solution path – the final solution is approached as $\Delta t \to \infty$. This property constitutes the regularization characteristics of the viscous approach that can significantly improve the global convergence behavior.

2 EXAMPLES

The concept that has been outlined in the previous sections was implemented into the commercial geotechnical finite element program TALPA[2]. Subsequent numerical simulations were carried out using TALPA.

2.1 *Simple 1D example with analytical solution*

It is illustrative to investigate the one-dimensional case a little more detailed. Assume a previously axially compressed specimen, being subjected to unloading at time $t = 0$ under constant stress conditions, i.e. ${}^{\infty}\varepsilon_q = const$. The time evolution of the axial swelling strain is then described by the relation

$$\varepsilon_q(t) = \int_0^t \dot{\varepsilon}_q \; d\tau = \int_0^t \frac{1}{\eta}\left({}^{\infty}\varepsilon_q - \varepsilon_q\right) \; d\tau \tag{10}$$

with the analytical solution

$$\varepsilon_q(t) = {}^{\infty}\varepsilon_q\left(1 - e^{-t/\eta}\right) \tag{11}$$

A FE-simulation with corresponding boundary conditions shows good accordance with the theoretical solution. An important aspect is, that – regardless of the time-step choice[3] – the solution paths converge to the stationary solution ${}^{\infty}\varepsilon_q$ as $t/\eta \to \infty$, affirming the observation that we have already made in the previous section (→ equations (9)). Note, that for the intermediate range, however, the time-step size naturally affects the accuracy of the solution.

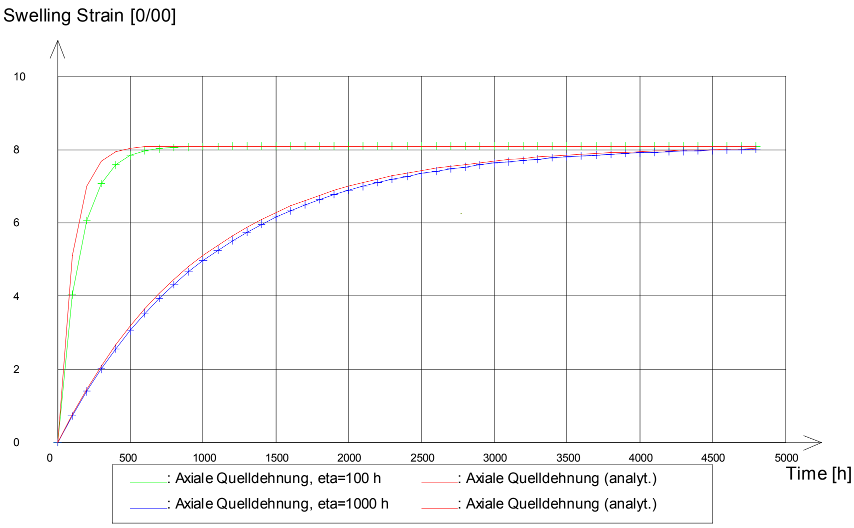

Figure 3. Plot of analytical vs. computed solution curves for different retardation times η

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[2] SOFiSTiK AG, Oberschleißheim, Germany

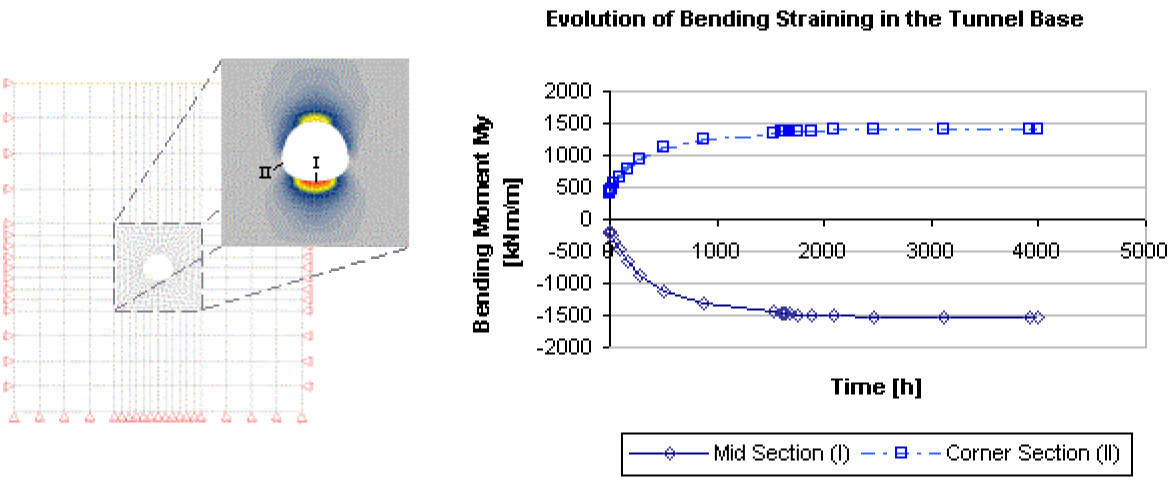
[3] Due to the implicit formulation the algorithm has the advantage of being unconditionally stable, i.e. there are no constraints to the time step size with respect to numerical oscillations.

2.2 *Tunnel simulation*

In a further – more real life engineering – example, a tunnel excavation process in swellable bedrock is simulated. After the lining is installed (time $t = 0$) a swelling analysis is performed.

Figure 4a shows the regions that ‘unload’ during the excavation process and where in consequence swelling strains develop with time. Due to the restraint provided by the tunnel lining significant compression states emerge and lead to a remarkable increase in the lining’s loading, the bending straining evolution of two representative cross sections in the tunnel base is plotted in Figure 4b. For the analysis a retardation time $\eta = 500\,h$ and a swelling modulus $k_q = 3.0$ ‰ is used.

Comparative calculations with the stationary (non-viscous) approach illustrate, that the viscous extension actually stabilizes the solution process, i.e. it turns out to be less sensitive to numerical problems.



(a) (b)

Figure 4. Volumetric swelling strain distribution for the final (stationary) state (a) and evolution of tunnel base’s bending straining (b)

3 CONCLUSIVE REMARKS

In this paper, an alternative procedure for the simluation of soil swelling effects is proposed. The procedure is based on an implicit continuum constitutive formulation within a viscous framework. From this approach, an algorithmic formulation that can be adopted within a finite element solution process is derived.

Illustrative finite element computations that have been carried out using the program TALPA[2], in which the proposed algorithm has been implemented, show that the implicit viscous extension is capable of simulating the time-dependant swelling characteristics appropriately.

As an important aspect, it is illustrated and proved that for $t/\eta \to \infty$ the time dependant solution path converges to the solution of the stationary problem. This property constitutes the regularization characteristics of the viscous approach. Comparative computations suggest use of the viscous extension also for stationary problems, since it can significantly improve the global convergence behavior.

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