Design for Steel Sections of Class 4

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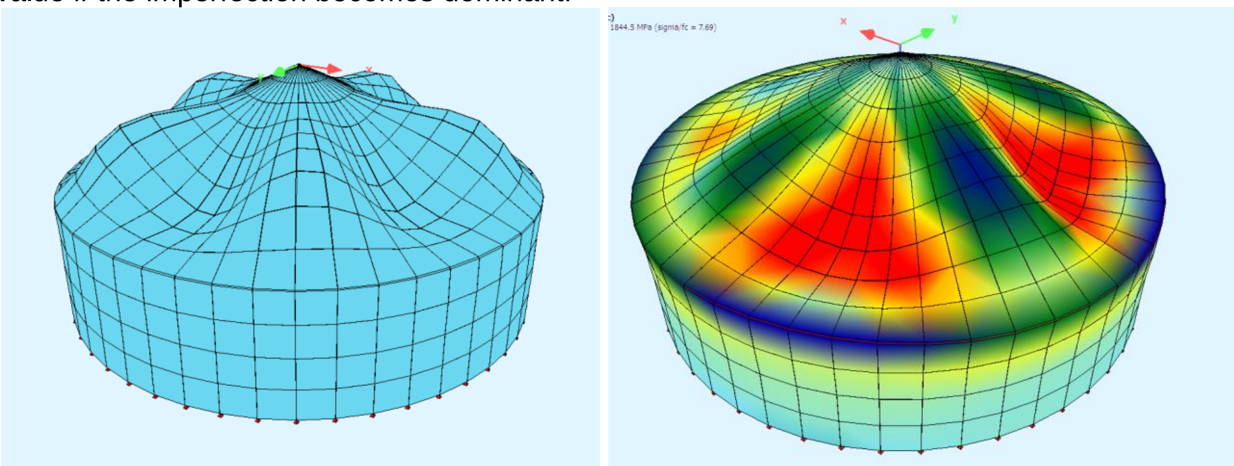
Summary

Thin plates may show buckling instabilities. The effort for a design check may become quite high. That is why simplified checks based on slenderness ratios (c/t) have a considerable impact on daily design tasks. The sections of class 4 defined in the Eurocode extend the simplified possibilities even further.

1. Buckling of plates

1.1 Structural Behaviour

A general problem for any stability problem is given by the fact that the buckling Eigen value is only weakly related to the ultimate load. Sometimes the real ultimate load is below the Eigen value if the imperfection becomes dominant:



Picture 1: Buckling eigen value 1.65 left, Ultimate load factor 1.08 right.

In many cases however there is a significant post buckling stabilization, the ultimate loading is well beyond the Eigen value:

Picture 2: Post Buckling behaviour of a web [6]

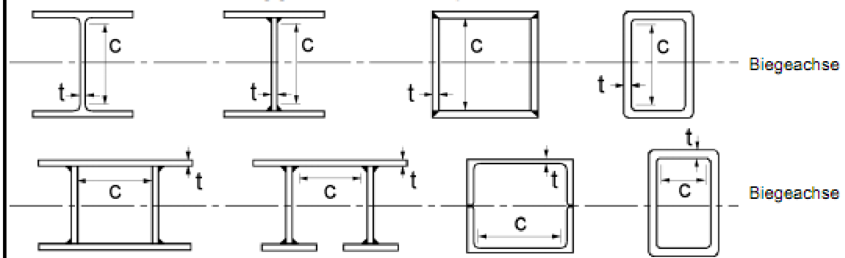
1.2 Limit Slenderness

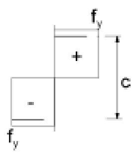
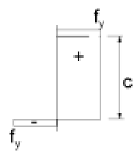
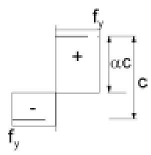
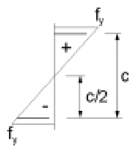
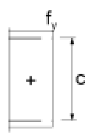
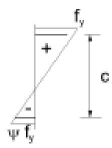
Nearly all design codes allow a stability check by slenderness limits, specified for four different classes. (see EN 1993-1-1, 5.5):

- Class 1 cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of resistance.
- Class 2 cross-sections are those which can develop their plastic moment resistance but have rotation capacity because of local buckling.
- Class 3 cross-sections are those in which the stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance.
- Class 4 cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross-section.

Tabelle 5.2 — Maximales c/t-Verhältnis druckbeanspruchter Querschnittsteile

Beidseitig gestützte druckbeanspruchte Querschnittsteile



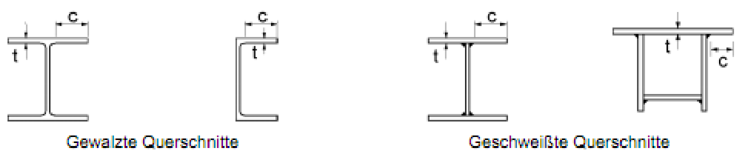
Klasse	auf Biegung beanspruchte Querschnittsteile	auf Druck beanspruchte Querschnittsteile	auf Druck und Biegung beanspruchte Querschnittsteile
Spannungsverteilung über Querschnittsteile (Druck positiv)			
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$	für $\alpha > 0{,}5$: $c/t \le \frac{396\varepsilon}{13\alpha - 1}$ für $\alpha \le 0{,}5$: $c/t \le \frac{36\varepsilon}{\alpha}$
2	$c/t \le 83\varepsilon$	$c/t \le 38\varepsilon$	für $\alpha > 0{,}5$: $c/t \le \frac{456\varepsilon}{13\alpha - 1}$ für $\alpha \le 0{,}5$: $c/t \le \frac{41{,}5\varepsilon}{\alpha}$
Spannungsverteilung über Querschnittsteile (Druck positiv)			
3	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	für $\psi > -1$: $c/t \le \frac{42\varepsilon}{0{,}67 + 0{,}33\psi}$ für $\psi \le -1$[a]: $c/t \le 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

$\varepsilon = \sqrt{235/f_y}$						
	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

[a] Es gilt $\psi \le -1$ falls entweder die Druckspannungen $\sigma \le f_y$ oder die Dehnungen infolge Zug $\varepsilon_y > \frac{f_y}{E}$ sind.

Tabelle 5.2 *(fortgesetzt)* — **Maximales** c/t**-Verhältnis druckbeanspruchter Querschnittsteile**

Einseitig gestützte Flansche



Klasse	auf Druck beanspruchte Querschnittsteile	auf Druck und Biegung beanspruchte Querschnittsteile: freier Rand im Druckbereich	auf Druck und Biegung beanspruchte Querschnittsteile: freier Rand im Zugbereich
Spannungs-verteilung über Querschnittsteile (Druck positiv)	+ c	αc + − c	αc + − c
1	$c/t \le 9\varepsilon$	$c/t \le \frac{9\varepsilon}{\alpha}$	$c/t \le \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \le 10\varepsilon$	$c/t \le \frac{10\varepsilon}{\alpha}$	$c/t \le \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Spannungs-verteilung über Querschnittsteile (Druck positiv)	+ c	+ − c	+ − c
3	$c/t \le 14\varepsilon$	$c/t \le 21\varepsilon\sqrt{k_\sigma}$ Für k_σ siehe EN 1993-1-5	

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

Many sectional tables specify sectional classes for individual sections. However a simple answer is not possible as these sectional class is depending on the stress distribution and the material strength [5]:

Tabelle 3.1. b/t Verhältnisse und Querschnittsklassen von Walzprofilen der Reihe IPE nach DIN 1025

Profil							Querschnittsklasse (QK)		
Kurzzeichen	b/t Verhältnisse vorhanden						Fe 360/ S235 ε = 1,0	Fe 360/ S275 ε = 0,92	Fe 360/ S355 ε = 0,81
	Steg			Flansch			Beanspruchung Druck **N**	Beanspruchung Druck **N**	Beanspruchung Druck **N**
	d [mm]	tw [mm]	d/tw	c [mm]	tf [mm]	c/tf			
IPE300	248	7,1	34,9	75	10,7	7,0	QK 2	QK 2	QK 4
IPE330	271	7,5	36,1	80	11,5	7,0	QK 2	QK 3	QK 4
IPE360	298	8,0	37,3	85	12,7	6,7	QK 2	QK 3	QK 4
IPE400	331	8,6	38,5	90	13,5	6,7	QK 3	QK 3	QK 4
IPE450	378	9,4	40,2	95	14,6	6,5	QK 3	QK 4	QK 4
IPE500	426	10,2	41,8	100	16,0	6,3	QK 3	QK 4	QK 4
IPE550	467	11,1	42,1	105	17,2	6,1	QK 4	QK 4	QK 4
IPE600	514	12,0	42,8	110	19,0	5,8	QK 4	QK 4	QK 4

More over it is an important question, if any type of load, especially fatigue loads should be allowed to mobilize plastic deformations. The design check has to be performed the other way round:

The user selects his design check first:

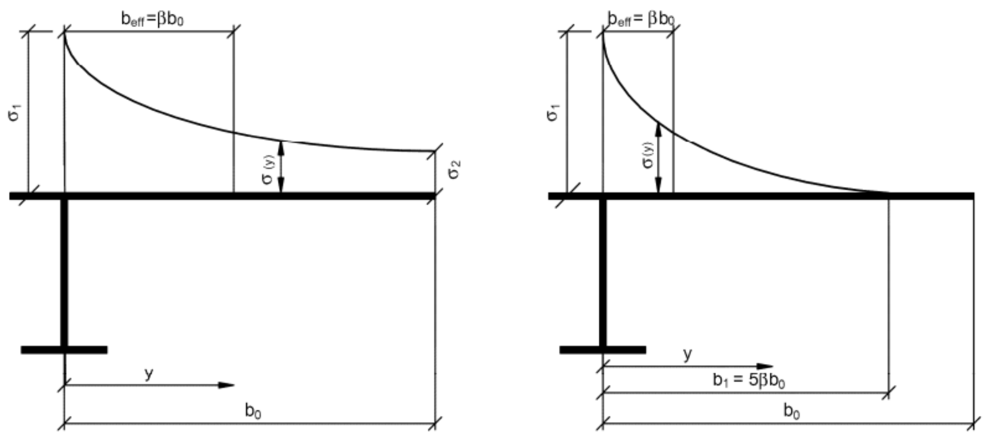
- elastic – elastic (= Stress calculation)
- elastic – plastic (= Usage of fully plasticized limits with interaction formulas)
- elastic – plastic (= Usage of nonlinear analysis within a section using true interaction)
- plastic – plastic (= Iteration of a fully nonlinear analysis)

Only then the slenderness limits c/t required for that particular check are evaluated. If the yield stress is not reached, larger values for the slenderness, those of the sectional class 3 may be used. The user will be bothered only for those cases where the check is not passed. And then he can select what to do in particular.

For very slender sections, exceeding the limits of class 3, there would be no allowance according to EN 1993-1-1 without switching to the non effective width according to EN 1993-1-5 provided in paragraph 5.5.2 (2). However this increases the effort to define sections considerably. Within the same paragraph, phrase (9) allows an increase of the limits as in DIN 18800 for small stresses. A third method is specified in EN 1993-1-5 chapter 10, but this method of reduced stresses does not allow for any redistribution of loading and is not economically in general and will not be discussed here.

Interestingly this sentence (9) requires the stresses to be obtained with 2nd order Theory if necessary, but does exclude the method for stability check according to chapter 6.3. Second order theory is favourable for tensile members, it is also clear that the calculation of deformations is difficult with fully plasticized sections, but the chapter 6.3.1.1. contains the important hint about the usage of non effective parts of the section for the stability analysis.

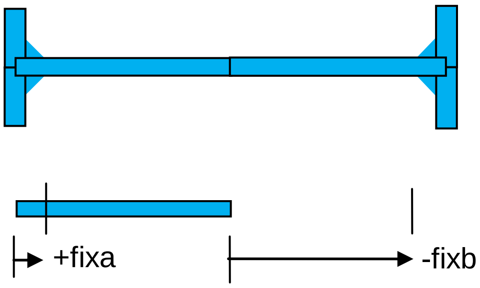
Chapter 3.2. of EN 1993-1-5 treats the influence of shear deformations via effective width for the serviceability. A nonlinear quartic distribution of the stresses is replaced by a simplified effective width without any normal stresses in the non effective area.



2. Slenderness limits with SOFiSTiK

2.1 Sectional definition

AQUA describes the element PLATE as thin walled structure, where two input values define the location of the transverse support. The general mechanism allows to subdivide each plate. Thus an array of plates may use the same definition of the span width.:



The location of those transverse supports may be specified explicitly by the user, but the program may select its own values based on the designation and orientation of the plates.

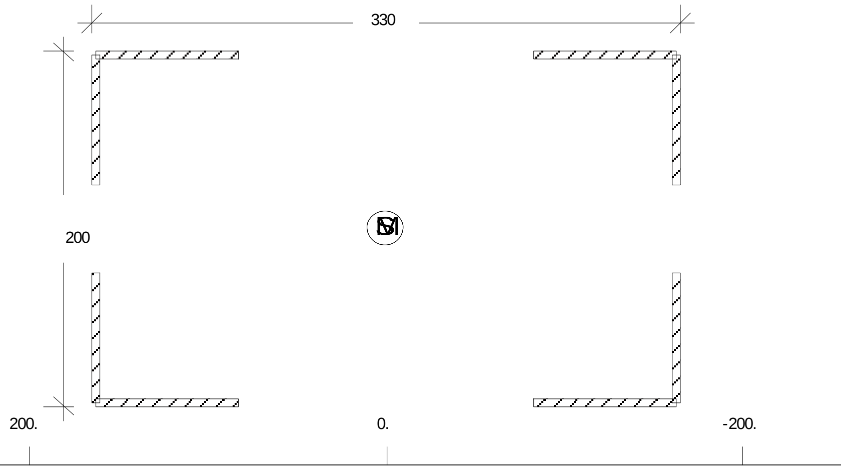
A similar mechanism is available for polygonal defined sections, which is the default due to higher precision of sectional areas. The slenderness has to be defined then by two stress points with an identical designation. The distance of those points define a c-value, the t-value is specified as an additional property. This construct allows a design for sectional classes 1 to 3 as well as the simplified increase of the limits according to sentence (9).

Stahlbaukalender 2000 [4] provides values for the ultimate bearing capacity of a hollow column. The value NIPL of the following table has been obtained by a nonlinear analysis which has been claimed to be close to the experimental results:

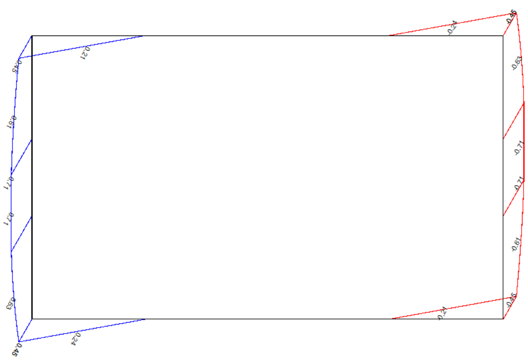
DIN 18800 Teil 3 Methode der reduzierten Spannungen	DIN 18800 Teil 2 Methode der wirksamen Breite	NIPL
$N_{Ed} = 284$ kN	$N_{Ed} = 363$ kN	$N_{Ed} = 451$ kN

The user may define his section according to the class 4 with non effective plate elements or the welding element. Both elements have the property to have no longitudinal stress, but shear stress. Small deviations are obtained in the graphical display and the treatment of the allowable stresses only.

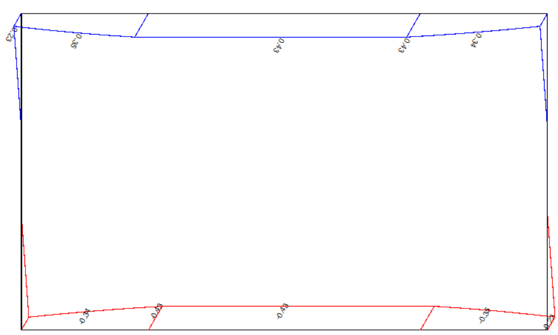
```
NORM EN 1993-2005
STEE 1 S 355
STO#B 326,157.5 ; LET#Y #B/2,(#B-#B(1))/2
STO#H 196,142.8 ; LET#Z #H/2,(#H-#H(1))/2
SECT 2 BEZ 'EFFECT. SECTION'
PLAT 11 -#Y    -#Z     -#Y(1)  -#Z    T  4
PLAT 12 -#Y(1) -#Z     +#Y(1)  -#Z    T  4 TYP NEFF
PLAT 13 +#Y(1) -#Z     +#Y     -#Z    T  4
PLAT 21 +#Y    -#Z     +#Y     -#Z(1) T  4
PLAT 22 +#Y    -#Z(1)  +#Y     +#Z(1) T  4 TYP NEFF
PLAT 23 +#Y    +#Z(1)  +#Y     +#Z    T  4
PLAT 31 +#Y    +#Z     +#Y(1)  +#Z    T  4
PLAT 32 +#Y(1) +#Z     -#Y(1)  +#Z    T  4 TYP NEFF
PLAT 33 -#Y(1) +#Z     -#Y     +#Z    T  4
PLAT 41 -#Y    +#Z     -#Y     +#Z(1) T  4
PLAT 42 -#Y    +#Z(1)  -#Y     -#Z(1) T  4 TYP NEFF
PLAT 43 -#Y    -#Z(1)  -#Y     -#Z    T  4
```



A section defined like this is still connected for shear, but the normal force is zero in the non effective parts. The equilibrium of stresses enforces a constant value of the shear in those areas.



Shear Vz (non effective)

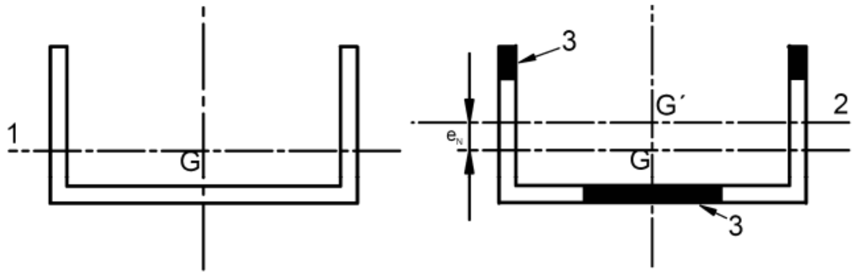


Shear Vy (non effective)

2.2 Application of slenderness limits in AQB

For the standard sectional check (elastic or plastic) the slenderness limits will be checked for thin walled and polygonal sections via the ratio of c/t and the limit values of table 5.2. Sections of class 4 may be accounted for by an increase of the limits according phrase (9).

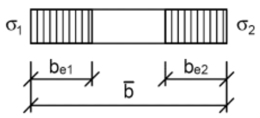
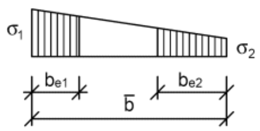
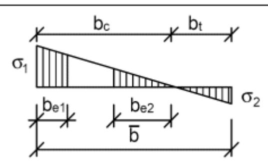
But if a nonlinear analysis is performed, it is possible to account for the provisions of EN 1993-1-5 for thin plates directly.



Bruttoquerschnitt   Wirksamer Querschnitt

The stresses in the two support points define the effective and non effective parts of every plate according to table 4.1:

Tabelle 4.1 — Zweiseitig gestützte druckbeanspruchte Querschnittsteile

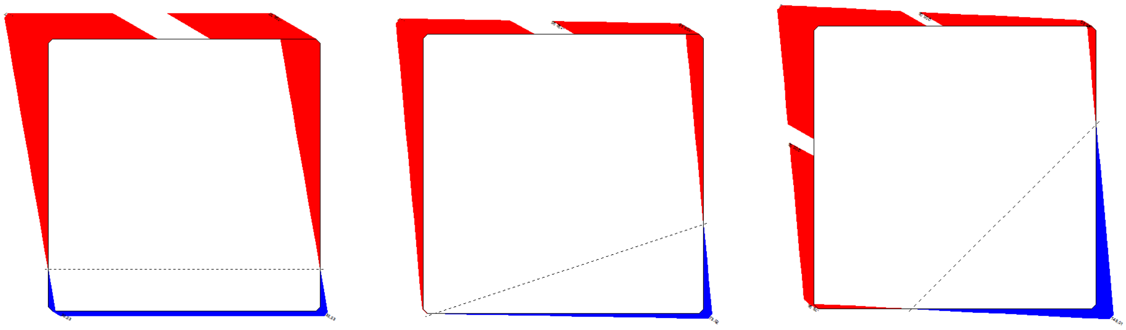
Spannungsverteilung (Druck positiv)	Wirksame Breite b_{eff}
	$\psi = 1$: $b_{eff} = \rho\, \bar{b}$ $b_{e1} = 0{,}5\, b_{eff}$ $b_{e2} = 0{,}5\, b_{eff}$
	$1 > \psi \geq 0$: $b_{eff} = \rho\, \bar{b}$ $b_{e1} = \frac{2}{5-\psi} b_{eff}$ $b_{e2} = b_{eff} - b_{e1}$
	$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, \bar{b}/(1-\psi)$ $b_{e1} = 0{,}4\, b_{eff}$ $b_{e2} = 0{,}6\, b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Beulwert k_σ	4,0	$8{,}2/(1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\,\psi + 9{,}78\,\psi^2$	23,9	$5{,}98\,(1-\psi)^2$

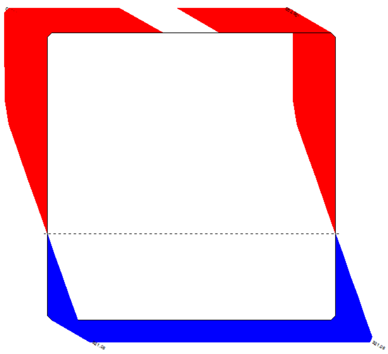
The reduction factor ρ is obtained by equations 4.2 resp. 4.3. The acting stress level may be accounted for with equation 4.4! It should be noted, that EN 1999 (Aluminium) defines only a reduction of the width of the thickness.

2.3 Example

A hollow section SH 800 x 800 x 16 made with S 355 exceeds with c/t = 47.5 the limits of the sectional class 3 calculated for the compressive part as 42.0·0.814 = 34.2. Thus this plate with a total length of 760 mm and a slenderness of λ_p=1.028 will be effective only within a width of 604 mm. Biaxial bending will shift the non effective parts to the most compressive corner:



As this reduction is not complete with increasing stresses, the question may be posed if it may be possible to go into the plastic region with the reduced section:



Then we have a rather complex logic of the program:

- In a first step all limits for c/t depending on the stress distribution and the effective stress are evaluated.
- If the yield stress is not reached, the section is classified as class 3 or 4.
- If the yield stress is reached, depending on the design task the limits of class 1 or 2 are checked. If these limits are not fulfilled and a nonlinear analysis is performed, the section will be reassigned to section class 4.
- If the section is in class 4, either the enlarged limits are checked for the stress analysis or the nonlinear design with dynamically selected non effective regions is obtained..

The printout of the c/t-values mirrors these variants:

`c/t=     27.26  < 42.0* 1.39(4)`	The limit is not exceeded
`c/t=     47.53 !< 42.0* 1.39(4)`	Limit is exceeded, the check is not passed
`c/t=     47.53  > 42.0* 1.09(4)` `  L=     760.5  beff=     604.3`	Limit is exceeded effective Length of plate has been applied

2.4 Assessment

The concept of the effective width has been developed in DASt-Richtlinie 016 for cold deformed shapes. With the Eurocode this nice approach has become available for hot rolled and welded shapes as well. The implementation in SOFiSTiK accounts for the dislocation of the center of gravity automatically. The strains will be varied in such a way that the inner forces and moments are in perfect equilibrium with the outer forces and moments.

In the definition of the effective width the critical buckling stress has been obtained including post critical effects. Similar reserve has been observed in the field of ship manufacturing.

2.5 Limits of applicably

Even EN 1993-1-5 defines limits of applicably. The definitions are valid only for nearly rectangular plates without essential holes ($d < 0.05b$). For elements with haunches e.g. non parallel flanges or webs without rectangular shapes, or members with irregular or regular larger holes may be analyzed according chapter 2.5 based on finite element models. Appendix C of EN 1993-1-5 has more hints for that subject.

If stiffeners are present, additional considerations are required. If Finite Elements are not used the stiffness of those stiffeners should be selected to allow the analysis as buckling plates. The interpolation between the buckling case and the behaviour as a beam requires the definition of the distance of the stiffeners in the direction of the beams, which is not available in general. Then there is chapter 5 of EN 1993-1-5 about shear buckling with imposes more problems in detail.

2.6 Interaction between shear an normal stress

The rules of chapter 7 of EN 1993-1-5 defines some very specific facts, but if we use the complete interaction based on the Prandtl yield criteria [3], no specific limits are expected. In a first step elastic normal- and shear stress is calculated, where the normal stress in the non effective regions is set to zero. Then the reduction factors for normal and shear stress are obtained based on the yield surface and the flow rule. The reduced stresses are integrated to total forces and moments.

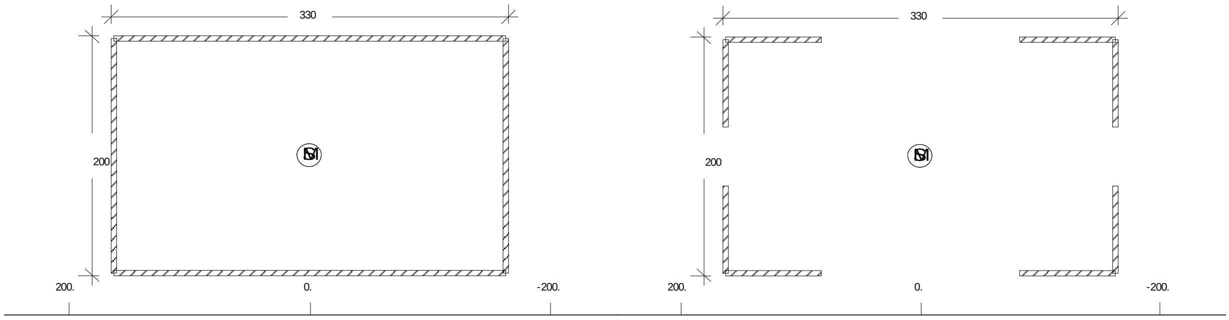
3. Example with check of system stability

We consider now the stability of a centric loaded hollow column made from S 355. The static system is a hinged beam with a length of 8.5 m. The section is a box with dimensions 200 x 330 x 4 mm. All plates belong to sectional class 4 and if we have a pure central loading we get:

$\lambda_P = (b/t) / (28{,}4 * \varepsilon * \sqrt{k_\sigma}) = 50 / (28{,}4 * 0.81 * \sqrt{4{,}0}) = 1{,}087$
$\rho = (\lambda_P - 0{,}22) / \lambda_P^2 = (1{,}087 – 0{,}22) / 1{,}087^2 = 0{,}734$
$beff = \rho * b = 0{,}734 * 200 = 146{,}8$ mm

$\lambda_P = (b/t) / (28{,}4 * \varepsilon * \sqrt{k_\sigma}) = 82.5 / (28{,}4 * 0.81 * \sqrt{4{,}0}) = 1{,}793$
$\rho = (\lambda_P - 0{,}22) / \lambda_P^2 = (1{,}793 – 0{,}22) / 1{,}793^2 = 0{,}489$
$beff = \rho * b = 0{,}489 * 330 = 161{,}5$ mm

and the full and an reduce effective section:



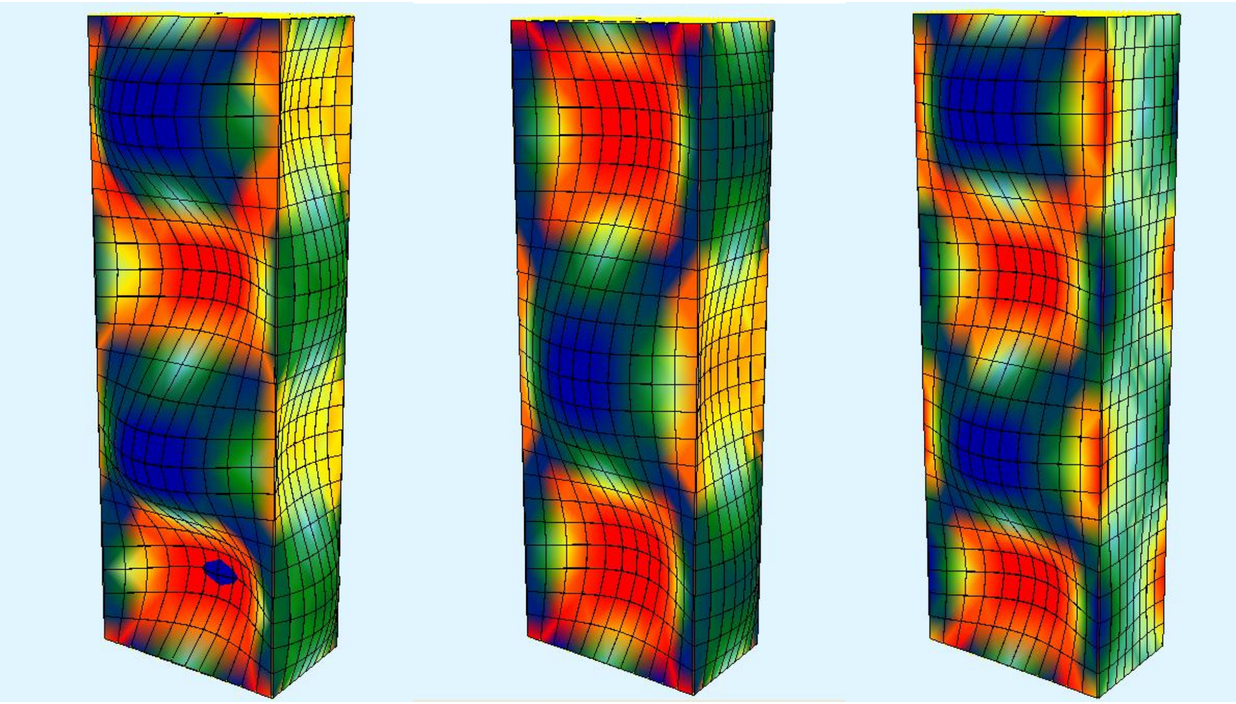

```
Sectional values
 Nr.    Mat      A[m2] Ay/Az/Ayz Iy/Iz/Iyz    ys/zs y/z-smp E/G-Modul      gam
        MBw     It[m4]      [m2]      [m4]     [mm]    [mm]   [N/mm2]   [kN/m]
   1      =   ORIG. SECTION
          1 4.1760E-03 2.353E-03 3.007E-05      0.0     0.0    210000     0.33
               6.259E-05 1.128E-03 6.476E-05    0.0     0.0     80769
   2      =   EFFECT. SECTION
          1 2.4024E-03 2.423E-03 1.702E-05      0.0     0.0    210000     0.19
               6.258E-05 1.362E-03 5.026E-05    0.0     0.0     80769
```

The central bearing capacity of this column with those sections is given (γ_{M1}=1.0):

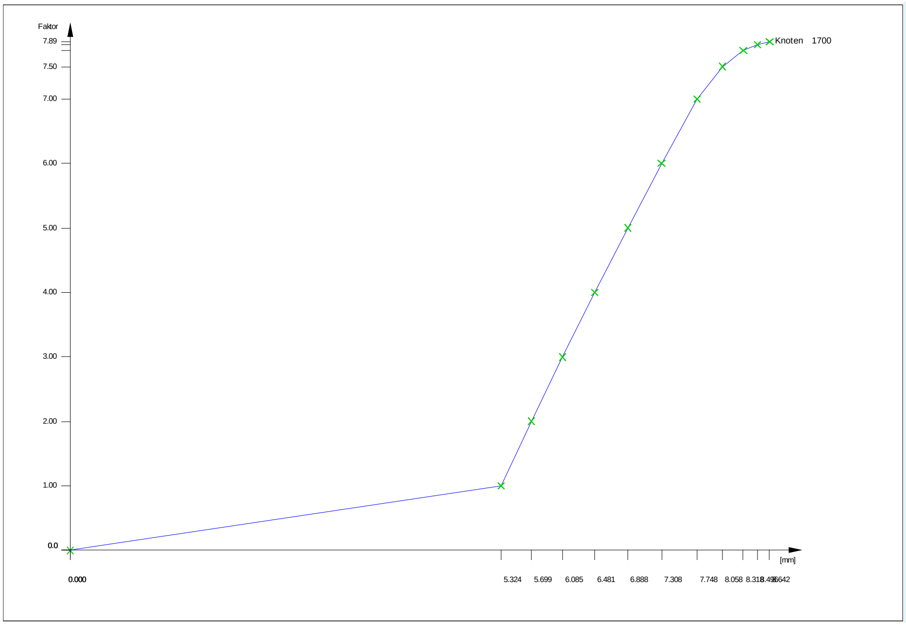
	S 1	S1 eff	S 2
Slenderness $\lambda = S_k/i$	100.2	100.2	101.0
Ideal buckling load	862.6	862.6	488.2
Calc. buckling Eigen value (12 elements)	854.4		486.1
Slenderness $\lambda = \sqrt{(A_{eff} f_y / N_{cr})}$	1.311	0.995	1.322
χ (Buckling curve b)	0.4216	0.601	0.4135
$N_{Ed} = \chi_z \ A_{eff} \ f_y \ / \ \gamma_{M1}$	624.8	512.6	352.7

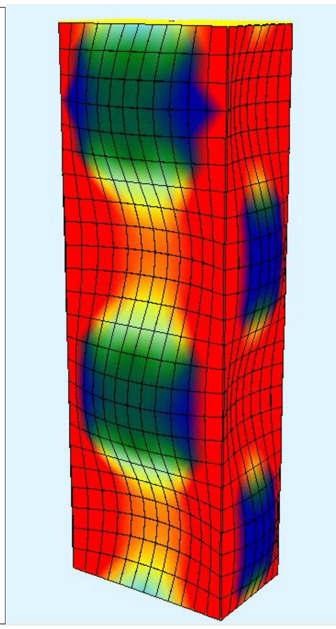
In a next step we calculate the ideal buckling eigen values with a FE-System of a short beam (1.0 m long).

3.1 Stability for FE-System

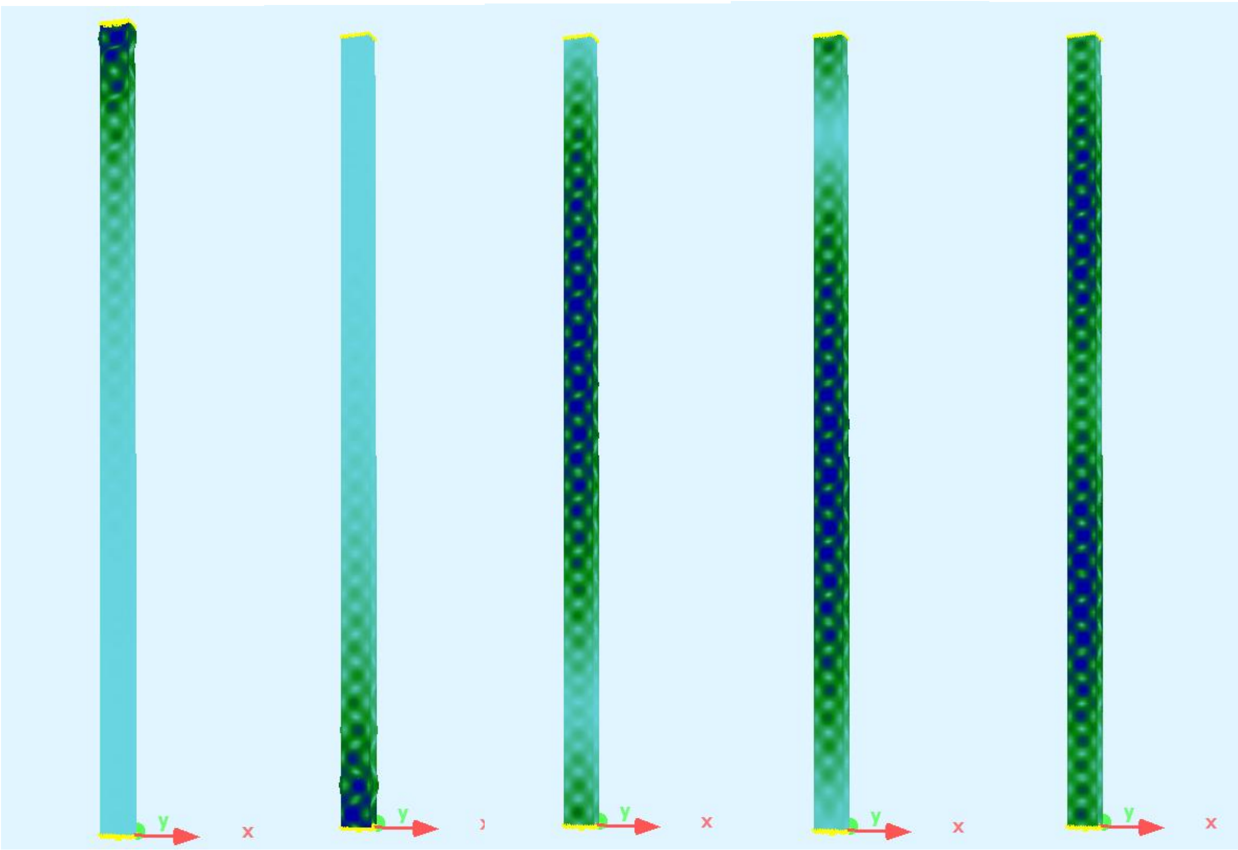


The Eigenvalues of the first three buckling forms are for a load of 590.9, 602.3 and 658.2 kN. If an ultimate limit analysis is performed with ASE using an imperfection of 5 mm from the first Eigenform, we get a value of 789 kN, we have some favourable post buckling effects.

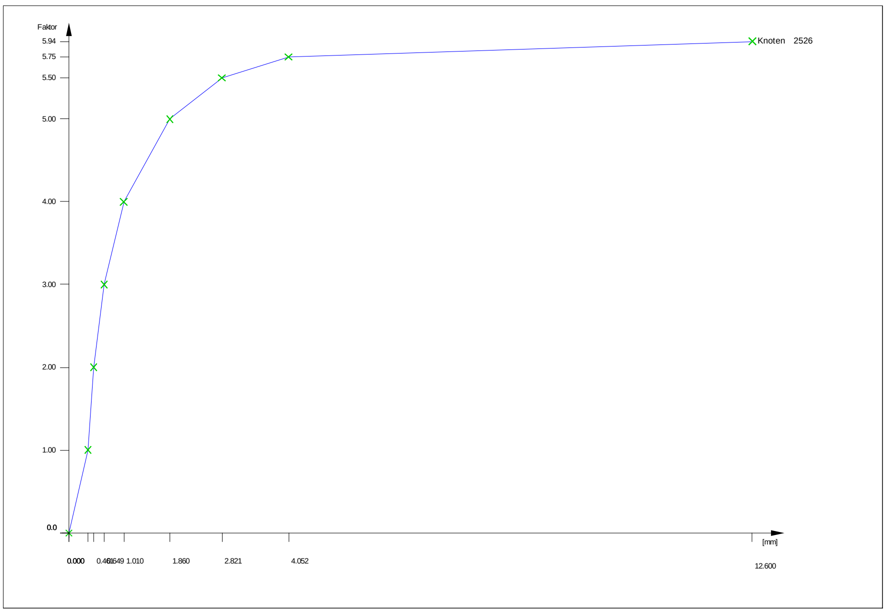


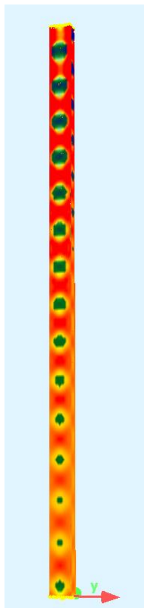


Switching to the total system with the length of 8500 mm, there are many local buckling shapes, the smallest two Eigenvalue are given with 587.9 kN and show the maximum buckles near the support.

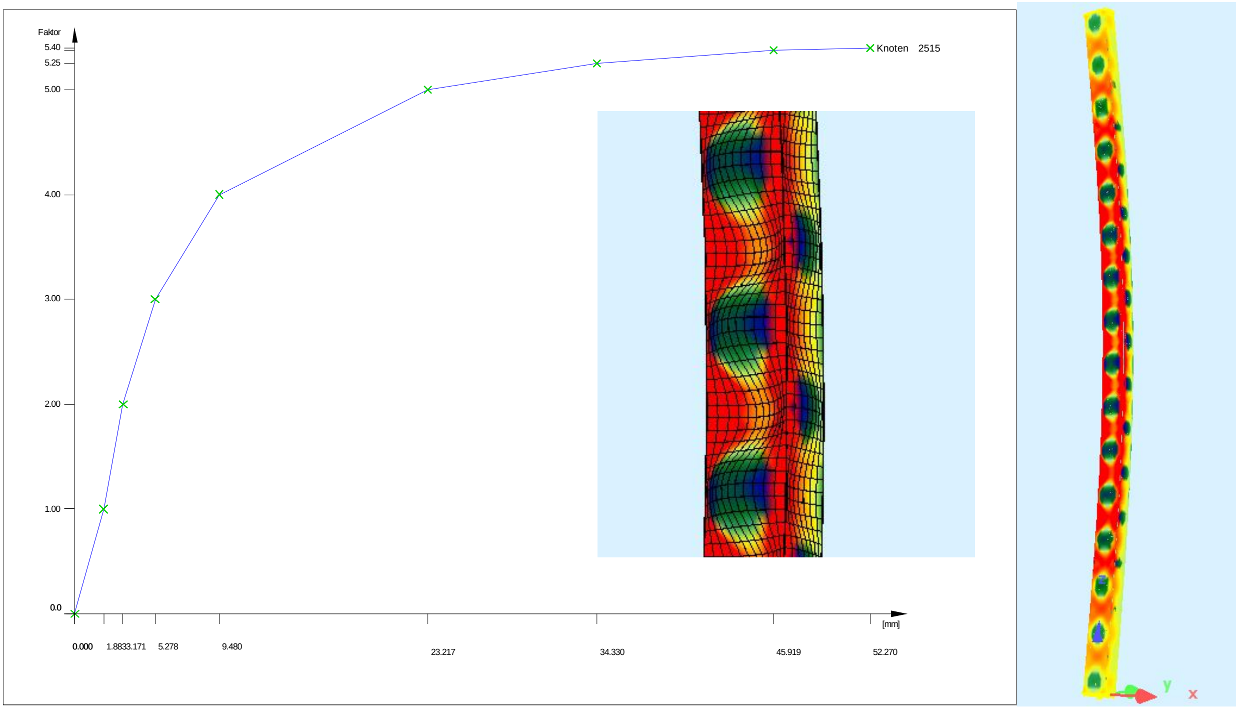


The ultimate limit load is now obtained to 594 kN, the initial imperfection has spread along the beam to the other end:





If we add now to the buckles with an imperfection of 1 mm a transverse loading equivalent for an imperfection of L/200 (curve b, plastic analysis), we get an ultimate load of 593.8 kN, but with a buckling imperfection of 5 mm only 472.3 kN:

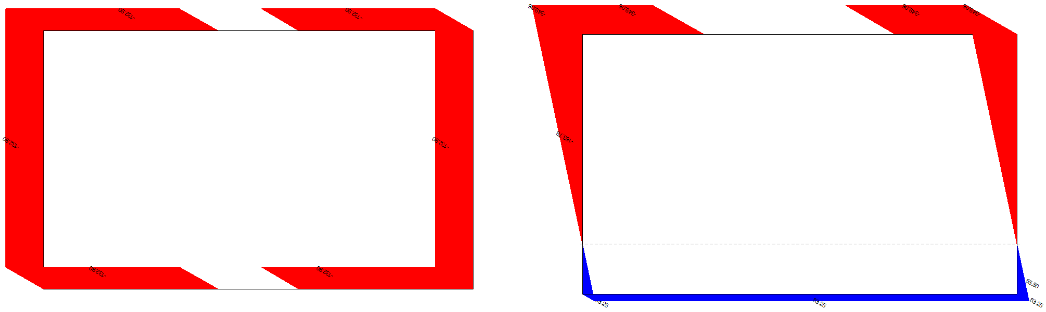


3.2 Stability with a beam system

As indicated from the table before, it is expected that a system with fixed ineffective regions will yield a smaller bearing capacity. The analysis showed an instable load of 431 kN, while 430 kN was still stable. This value is higher than the 320 kN, obtained from the buckling curve directly, but smaller than the FE bearing capacity of 594 kN. The bearing capacity of the automatic section was smaller however. A reason for that effect is the dislocation of the center of gravity, yielding an extra imperfection and increasing the distance to the compressive fibres.

This shows that the size of the imperfection is critical. While we have for cold deformed shapes buckling curve, c with 1/200 for elastic analysis, for hot rolled shapes we have curve a with 1/300. For very slender sections curve a may be discussed. This selection has a great influence. Due to the additional imperfection from the dislocation we selected an imperfection of only 1/500, yielding an ultimate load of 433 kN. The main usage of the proposed method will be for sections below the ultimate loading. Shortly before reaching the ultimate load we have:

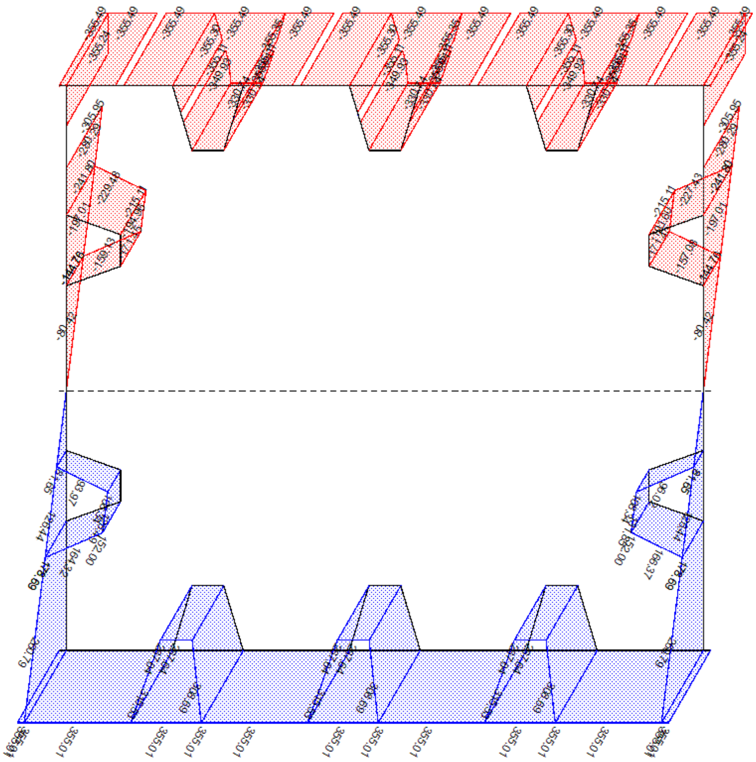
```
Beam     x[m]      e-o    ky/kz     x  zn/yn      Ni/Vi  Myi/Mzi  Ey/Ez/G-EFF
                 [o/oo]  [1/km]  [mm]  [mm]       [kN]    [kNm]     [N/mm2]
1001    0.000  -0.568    0.000 379.8    -.-     -432.1     0.00      210000
             ---------------- plast.fact.  sig[MPa] tau[MPa]   eps[o/oo] sII[MPa]
             panel           1 -119.36  -119.36 c/t=     79.50  > 42.0*0.814(4)
                                  psi=      1.00   L=    318.0  beff=     248.5
             panel           2 -119.36  -119.36 c/t=     47.00  > 42.0*0.814(4)
                                  psi=      1.00   L=    188.0
             panel           3 -119.36  -119.36 c/t=     79.50  > 42.0*0.814(4)
                                  psi=      1.00   L=    318.0  beff=     248.5
             panel           4 -119.36  -119.36 c/t=     47.00  > 42.0*0.814(4)
                                  psi=      1.00   L=    188.0    1001    0.000
1006    0.000 -0.715    8.444 184.7  84.7     -432.1    34.22       134791
             ---------------- plast.fact.  sig[MPa] tau[MPa]   eps[o/oo] sII[MPa]
             panel           1 -323.90  -323.90 c/t=     79.50  > 42.0*0.814(4)
                                  psi=      1.00   L=    318.0  beff=     167.6
```



For the centric loaded end section the non effective parts on the short plates have vanished as the stresses are rather small. At the mid span the non effective region of the compressive zone has become quite large, while the tensile zone is fully effective. The yield strength is not yet reached, if this value is exceeded the system collapses rather immediately.

4. Bridgesection

In the German Stahlbaukalender 2009 [7] a bridge section is presented with a lot of ineffective plates. The automated system allows a very simple description and analysis of the section.



The loading in the picture above is 1.5 times higher than the given values in the example. For the original moment many of the ineffective regions do not need to be accounted for because the stress is not as high.

5. Conclusion

The automatic Iteration of non effective regions depending on the acting stress distribution allows the design of slender sections for all ranges of slenderness according to the Eurocode. The simplified reduced bearing capacity according to EN 1993-1-1 has been calculated with 513 kN, which is higher than the more detailed analysis according EN 1993-1-5 with 433 kN, but less than the true capacity of 594 kN. This load is highly sensitive on the assumed size of the imperfection.

6. Literature

[1] EN 1993-1-1, 2005

[2] EN 1993-1-5, 2007

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[5] Schürzinger, S. Sections der Klasse 4 und ihre Anwendung mit SOFiSTiK Diplomarbeit an der FH München – Stahlbau, 2003

[6] Hannebauer, D., Zur Querschnitts- und Stabtragfähigkeit von Trägern mit profilierten Stegen, Dissertation BTU Cottbus (2008)

[7] Braun, B., Kuhlmann, U., Bemessung und Konstruktion von aus Blechen zusammengesetzten Bauteilen nach DIN EN 1993-1-5, Stahlbaukalender 2009, Ernst & Sohn, Berlin (2009)